

## Globalization and Synchronization of Innovation Cycles

By

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# **Introduction**

### ***Theoretical Motivation:***

- How does globalization affect macro co-movements across countries?
- Most economists address this question by assuming that some *exogenous* processes drive productivity movements in each country.
- *But*, globalization (a trade cost reduction) can affect
  - productivity growth rates, as already shown by endogenous growth models
  - *synchronicity* of productivity fluctuations, as we show in an *endogenous cycle model*

### ***Empirical Motivation:***

- Countries that trade more with each other have more synchronized business cycles
  - Particularly among developed countries (EU, OECD, etc)
  - Not so between developed and developing countries
- Difficult to explain this “*trade-comovement puzzle*” in models with exogenous shocks
  - Common shocks would cause synchronization *regardless* of the trade intensity
  - With country-specific shocks, more trade lead to *less* synchronization
  - Attempts to resolve it by global supply chains met limited success; also awkward
- Easier in models of endogenous fluctuations. No need to appeal to global supply chains

### ***Intuition We Want to Capture***

- Two ***structurally identical*** countries
- Each country (in autarky) is subject to endogenous fluctuations, due to strategic complementarities in the ***timing*** of innovation among firms competing in the same market
- Without trade, fluctuations in the two countries are obviously ***disconnected***.
- Trade integration makes firms based in different countries compete against each other and respond to an increasingly global (hence common) market environment.
- Strategic complementarities in the ***timing*** of innovation across countries
- ***Even with partial integration***, this causes an alignment of innovation incentives, ***synchronizing*** innovation activities and productivity fluctuations across countries

### ***What We Do***

To capture this intuition in a simplest manner, we develop a 2-country model of endogenous innovation cycles with *two* building blocks

**Judd (1985; sec.4), also Deneckere & Judd (1992)**, for endogenous innovation cycles  
*Unique* equilibrium path exhibits fluctuation; which can be obtained by iterating a 1D-noninvertible PWL map, commonly called “a skew-tent map”

**Helpman & Krugman (1985; ch.10)** for intra-industry trade with iceberg trade cost, which is used as a *coupling* parameter.

Conceptually, this is a study of ***Synchronization of (Weakly) Coupled Oscillators***

## *Synchronization of (Weakly) Coupled Oscillators*

*Natural Science:* A Major Topic. Thousands of examples: Just to name a few,

- Christiaan Huygens' pendulum clocks
- The Moon's rotation and revolution
- London Millennium Bridge

Also, search videos "Synchronization of Metronomes" on Youtube!

But, we cannot use the existing studies of "coupled skew-tent maps."

- Without microfoundations, we cannot give structural interpretation to their "synchronizing forces" nor "coupling" parameters.
- Subject to the Lucas critique. In GE, innovation incentive might change with trade.

*Economics:* None? To the best of our knowledge, this is

- First 2-country, dynamic GE model of endogenous fluctuations
- Our companion piece, "Interdependent Innovation Cycles"
  - Two-sector, closed economy
  - Each sector produces a Dixit-Stiglitz composite, as in DJ
  - CES preferences over the two composites
  - Fluctuations in the two sectors are decoupled for Cobb-Douglas
  - Synchronized (asynchronized) if EoS increases (decreases) from one.

## **The Two Building Blocks**

**Judd (1985);** Dynamic Dixit-Stiglitz monopolistic competitive model; Innovators pay fixed cost to introduce a new (horizontally differentiated) variety

**Judd (1985; Sec.2);** They keep their monopoly power. Unique steady state globally stable due to *intertemporal smoothing* of innovation activities

*Main Question: What if they have monopoly for a limited time?*

- Each variety sold initially at monopoly price; later at competitive price
- Impact of an innovation, initially muted, reach its full potential *with a delay*
- Past innovation discourages innovators more than contemporaneous innovation
- **Temporal clustering of innovation**, leading to aggregate fluctuations

**Judd (1985; Sec.3);** *Continuous time* and monopoly lasting for  $0 < T < \infty$

- *Delayed differential equation* with an infinite D state space
- For  $T > T_c > 0$ , the economy oscillates between the phases of active innovation and of no innovation along any equilibrium path for almost all initial conditions.

**Judd (1985; Sec.4); also Deneckere & Judd (1992; DJ for short)**

- *Discrete time* and *one period monopoly* for analytical tractability
- **1D state space** (the measure of competitive varieties inherited from past innovation determines how saturated the market is)
- Unique equilibrium path generated by a **1D PWL noninvertible (i.e., skew-tent) map**
- When the unique steady state is unstable, converging to a **2-cycle** or to a *chaotic attractor* from almost all initial conditions.

## Deneckere-Judd (DJ) in a Nutshell

$n_t$ : (Measure of) competitive varieties inherited per labor supply

### Noninvertible PWL (Skew Tent) map

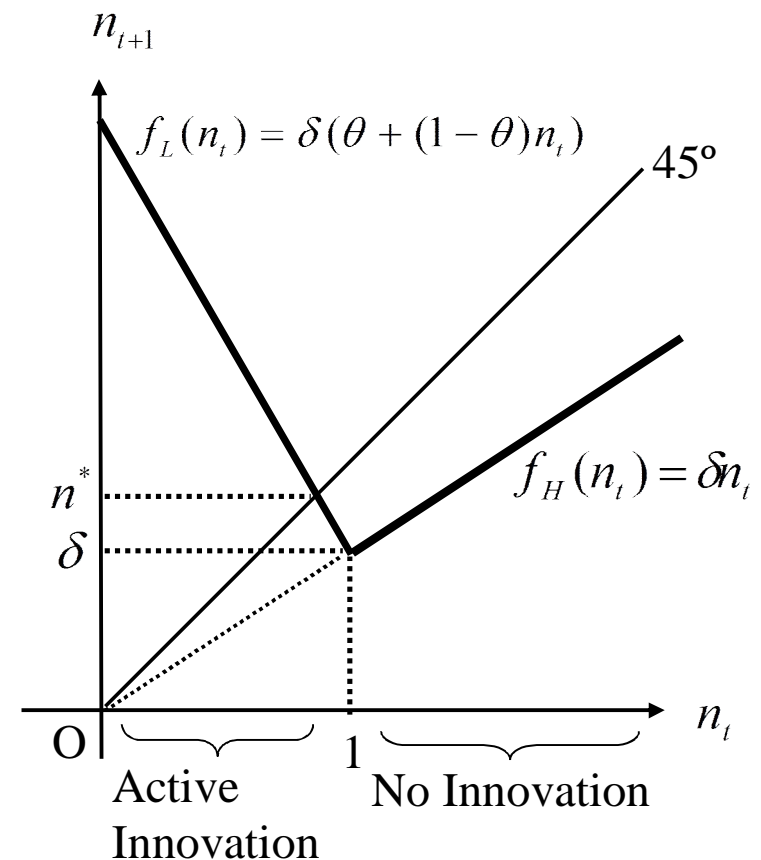
$$n_{t+1} = f(n_t) \equiv \begin{cases} f_L(n_t) \equiv \delta(\theta + (1-\theta)n_t) & \text{if } n_t < 1 \\ f_H(n_t) \equiv \delta n_t & \text{if } n_t > 1 \end{cases}$$

$\delta \in (0,1)$ , Survival rate of varieties due to obsolescence (or exogenous labor supply growth)

$$\theta \equiv \left(1 - \frac{1}{\sigma}\right)^{1-\sigma} \in (1, e), \text{ increasing in } \sigma \text{ (EoS)}$$

Market share of a competitive variety relative to a monopolistic variety

$\theta - 1 > 0$ : the delayed impact of innovations

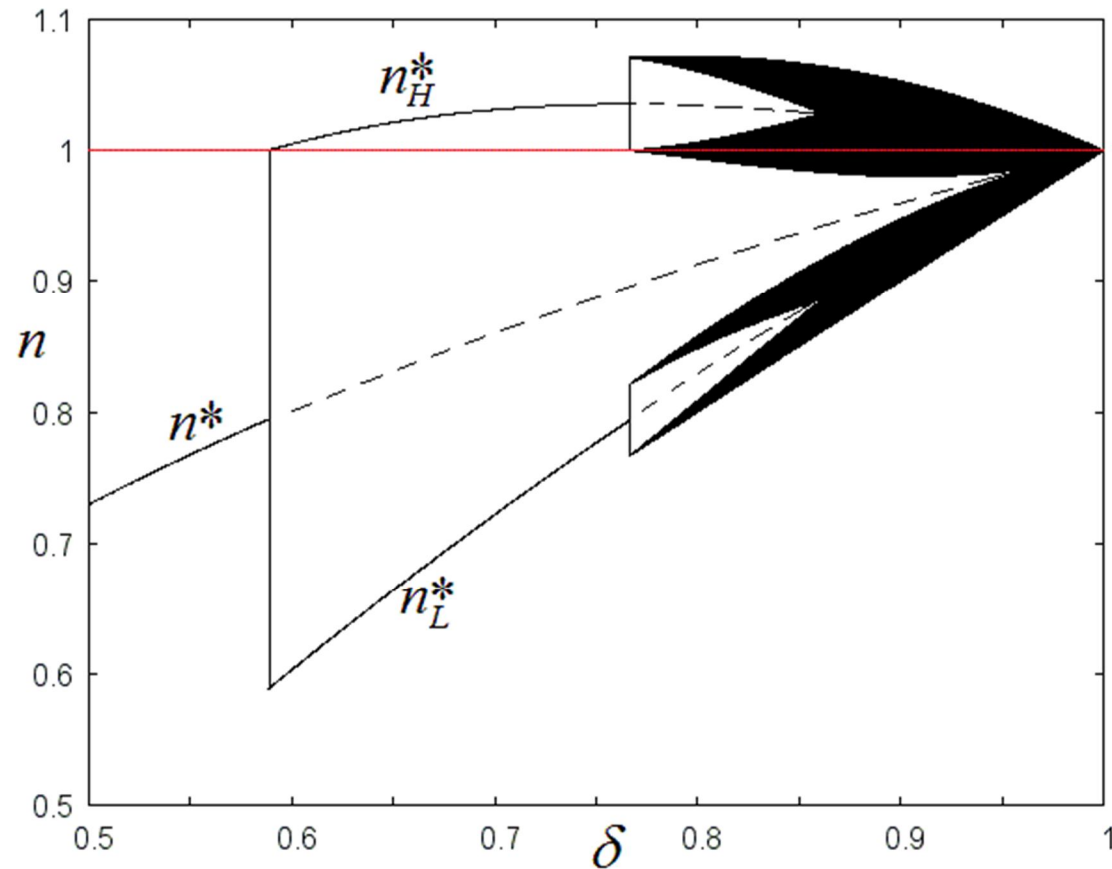




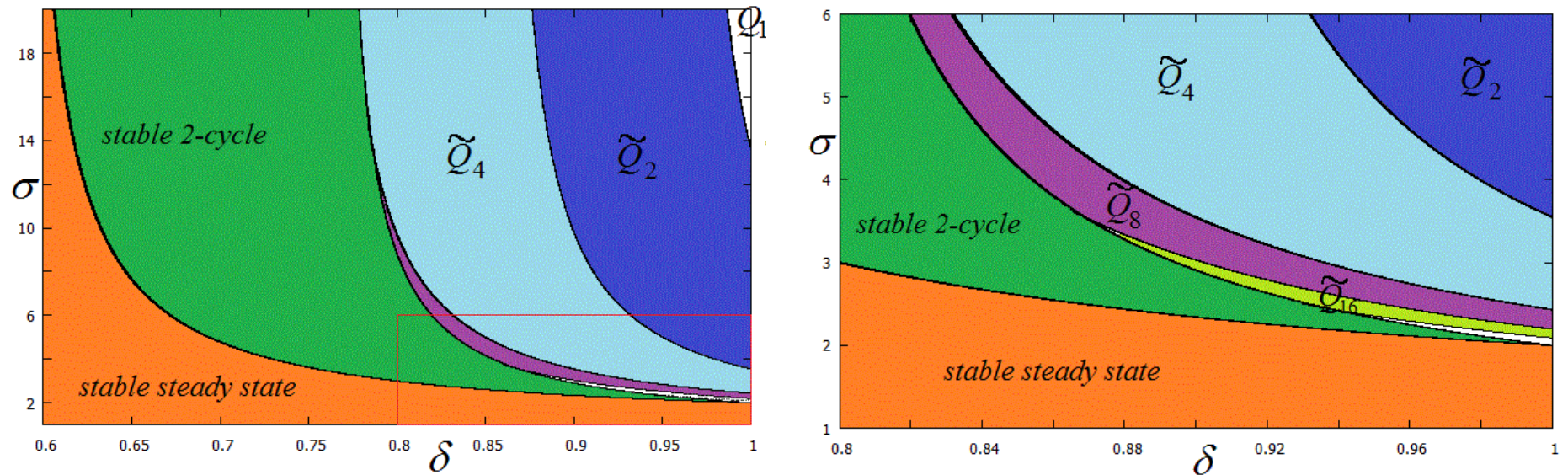
**A Unique Attractor:**

- Stable steady state for  $\delta(\theta - 1) < 1$
- Stable 2-cycle for  $\delta^2(\theta - 1) < 1 < \delta(\theta - 1)$
- Robust chaotic attractor (of various types) for  $\delta^2(\theta - 1) > 1$

**Effects of a higher  $\delta$**



## Bifurcation diagram in the $(\sigma, \delta)$ -plane and its magnification



Endogenous fluctuations with  
 a higher  $\sigma$  (more substitutable, strong incentive to avoid competition)  
 a higher  $\delta$  (more past innovation survives to crowd out current innovation).

We focus on **the stable 2-cycle case**,  $\delta^2(\theta - 1) < 1 < \delta(\theta - 1)$ .

*Helpman & Krugman (1985; Ch.10):*

Trade in horizontally differentiated (Dixit-Stiglitz) goods with *iceberg trade costs* between two *structurally identical* countries; only their sizes may be different.

- **In autarky**, the number of firms based in each country is proportional to its size.
- **As trade costs fall**,
  - Differentiated goods produced in the two countries mutually penetrate each other's home markets (Two-way flows of goods).
  - Firm distribution becomes increasingly skewed toward the larger country (*Home Market Effect and its Magnification*)

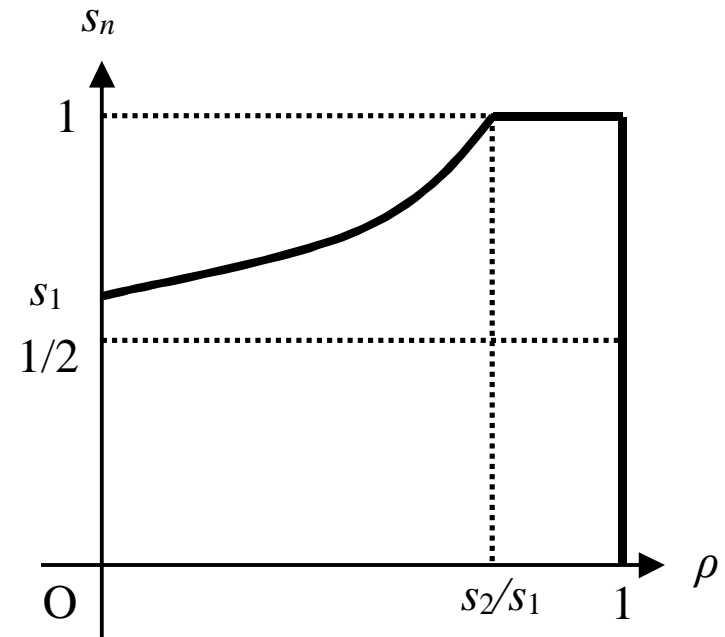
**Two Parameters:**  $s_1$  &  $\rho$

$s_1 = 1 - s_2 \in [1/2, 1)$ :

Bigger country's share in market size

$\rho \equiv (\tau)^{1-\sigma} \in [0, 1)$ : Degree of Globalization:  
*inversely related to the iceberg cost,  $1 < \tau \leq \infty$*

$s_n$ : Bigger country's share in firm distribution



# **A Two-Country Model of Endogenous Innovation Cycles**

**Our Main Results:** By combining DJ (1992) and HK (1985):

- **2D state space:** (Measures of competitive varieties in the two countries)
  - Unique equilibrium path obtained by iterating a **2D-PWS, noninvertible map** with *four parameters*:  $\theta$  &  $\delta$  &  $s_1$  &  $\rho$ 
    - One unit of competitive varieties =  $\theta$  ( $> 1$ ) units of monopolistic varieties
    - One unit of foreign varieties =  $\rho$  ( $< 1$ ) unit of domestic varieties
  - **In autarky** ( $\rho = 0$ ), the dynamics of the two are **decoupled**. Whether they converge to synchronized or asynchronous 2-cycles depends on how you draw the initial condition
  - **As trade costs fall** (a higher  $\rho$ ), they become more **synchronized**:
    - *Basin of attraction* for asynchronous 2-cycles **shrinks** and **disappears**
    - *Basin of attraction* for synchronized 2-cycles **expands**.
- Full synchronization occurs with partial trade integration (i.e.,  $\rho < 1$ )*
- Fully synchronized at a larger trade cost if country sizes are more unequal
  - Even a small size difference speeds up synchronization significantly
  - The larger country sets the tempo of global innovation cycles, with the smaller country adjusting its rhythm

**2D Dynamical System;**  $n_{t+1} = F(n_t)$  with  $n_t \equiv (n_{1t}, n_{2t}) \in R_+^2$ ;  
 $(0 < \delta < 1; 1 < \theta < e; 0 \leq \rho < 1; 1/2 \leq s_1 < 1)$

$$\begin{aligned} n_{1t+1} &= \delta(\theta s_1(\rho) + (1-\theta)n_{1t}) & \text{if } n_t \in D_{LL} &\equiv \{(n_1, n_2) \in R_+^2 \mid n_j \leq s_j(\rho)\} \\ n_{2t+1} &= \delta(\theta s_2(\rho) + (1-\theta)n_{2t}) \end{aligned}$$

$$\begin{aligned} n_{1t+1} &= \delta n_{1t} & \text{if } n_t \in D_{HH} &\equiv \{(n_1, n_2) \in R_+^2 \mid n_j \geq h_j(n_k)\} \\ n_{2t+1} &= \delta n_{2t} \end{aligned}$$

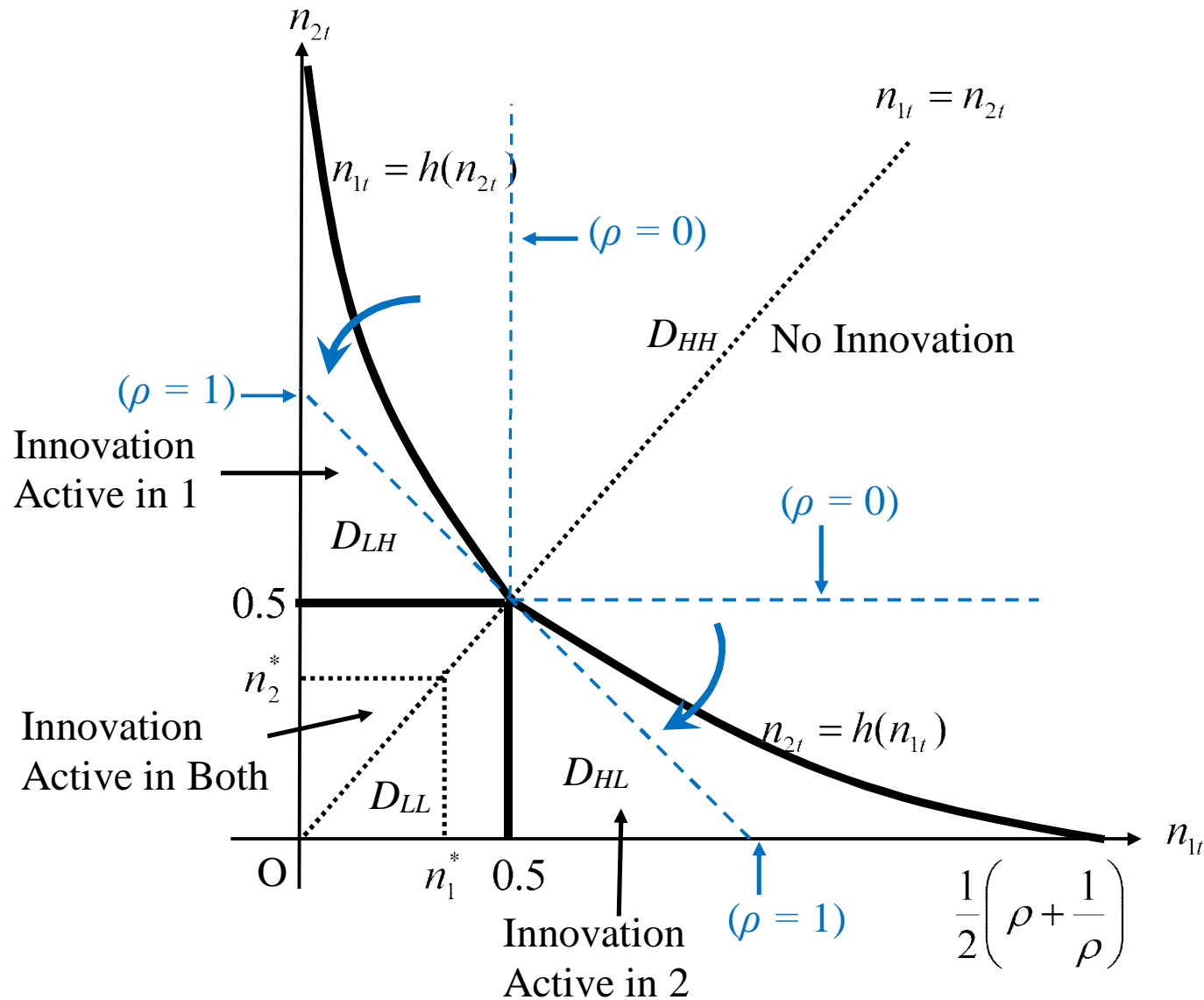
$$\begin{aligned} n_{1t+1} &= \delta n_{1t} & \text{if } n_t \in D_{HL} &\equiv \{(n_1, n_2) \in R_+^2 \mid n_1 \geq s_1(\rho); n_2 \leq h_2(n_1)\} \\ n_{2t+1} &= \delta(\theta h_2(n_{1t}) + (1-\theta)n_{2t}) \end{aligned}$$

$$\begin{aligned} n_{1t+1} &= \delta(\theta h_1(n_{2t}) + (1-\theta)n_{1t}) & \text{if } n_t \in D_{LH} &\equiv \{(n_1, n_2) \in R_+^2 \mid n_1 \leq h_1(n_2); n_2 \geq s_2(\rho)\} \\ n_{2t+1} &= \delta n_{2t} \end{aligned}$$

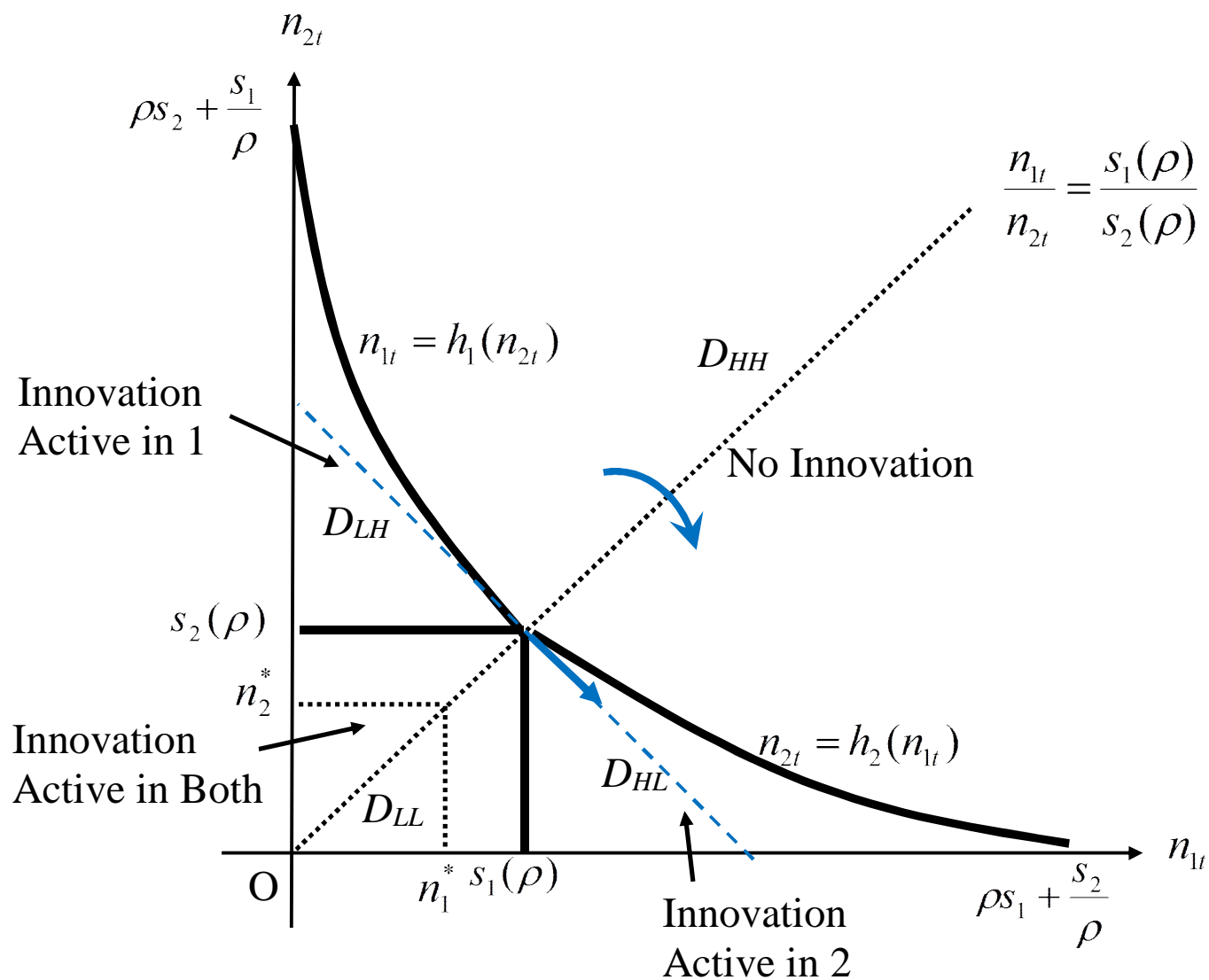
where  $s_1(\rho) = 1 - s_2(\rho) = \min\left\{\frac{s_1 - \rho s_2}{1 - \rho}, 1\right\}$ ,  $0.5 \leq s_1 = 1 - s_2 < 1$ ;

$h_j(n_k) > 0$  defined implicitly by  $\frac{s_j}{h_j(n_k) + \rho n_k} + \frac{s_k}{h_j(n_k) + n_k / \rho} = 1$ .

**State Space & Four Domains for the Symmetric Case:  $0 < \rho < s_2 / s_1 = 1$**



### State Space & Four Domains for the Asymmetric Case: $0 < \rho < s_2 / s_1 < 1$

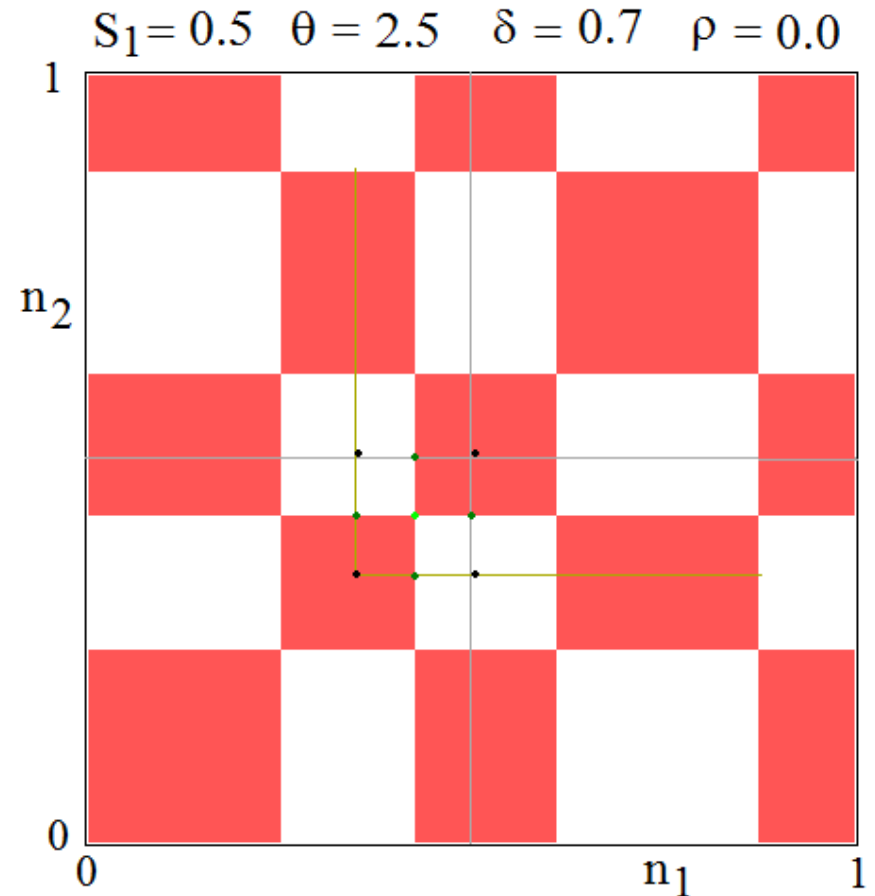




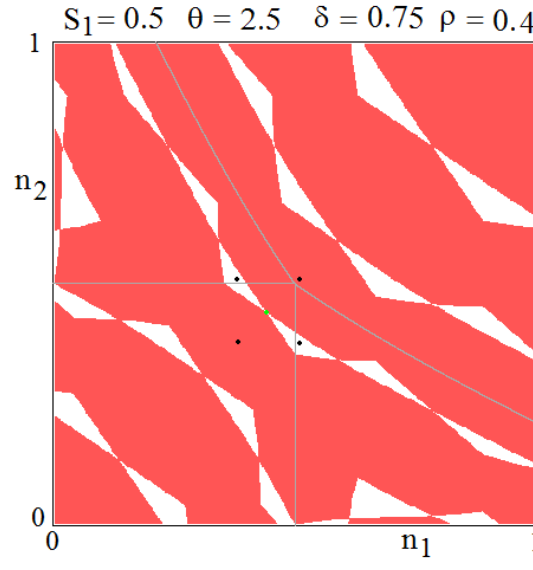
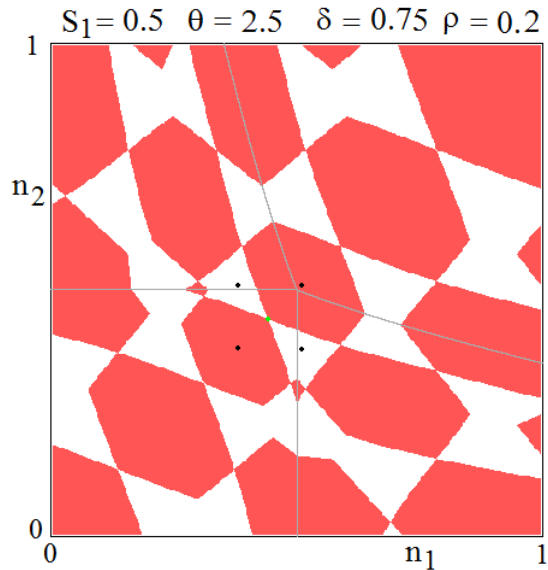
**Synchronized vs. Asynchronized 2-Cycles in Autarky:**  $\rho = 0; \delta(\theta - 1) > 1 > \delta^2(\theta - 1)$ ,

As a 2D-map, this system has

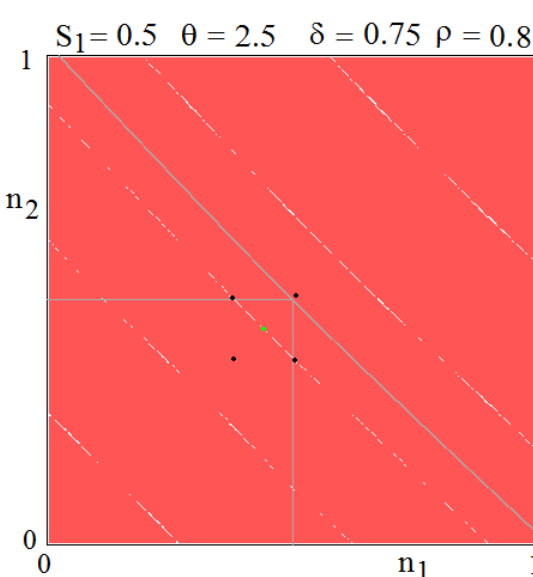
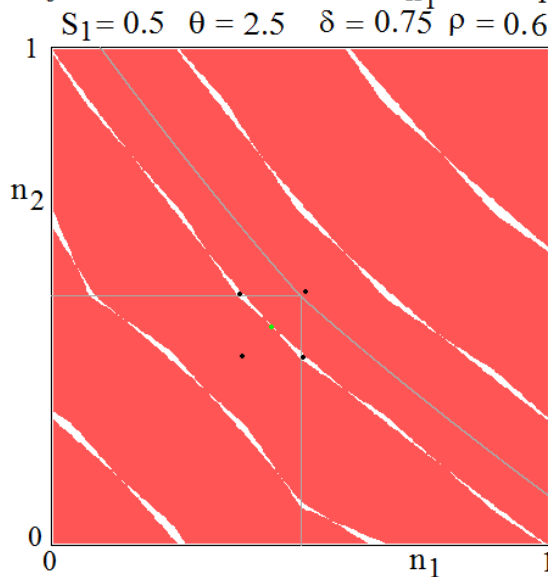
- **An unstable steady state;**  $(n_1^*, n_2^*)$
- **A pair of stable 2-cycles**
  - **Synchronized;**  $(n_{1L}^*, n_{2L}^*) \leftrightarrow (n_{1H}^*, n_{2H}^*)$ ,  
*Basin of Attraction in red.*
  - **Asynchronized;**  $(n_{1L}^*, n_{2H}^*) \leftrightarrow (n_{1H}^*, n_{2L}^*)$ ,  
*Basin of Attraction in white*
- **A pair of saddle 2-cycles:**  
 $(n_{1L}^*, n_2^*) \leftrightarrow (n_{1H}^*, n_2^*) ; (n_1^*, n_{2H}^*) \leftrightarrow (n_1^*, n_{2L}^*)$



**Symmetric Synchronized & Asynchronized 2-Cycles:  $s_1 = 0.5$ ;  $\theta = 2.5$ ;  $\delta = 0.75$**

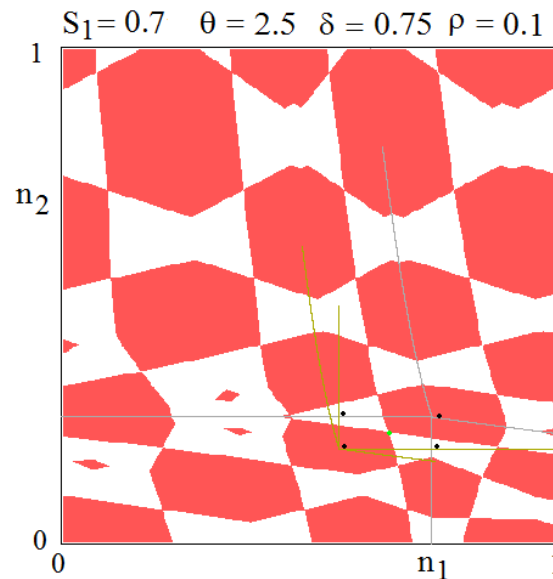
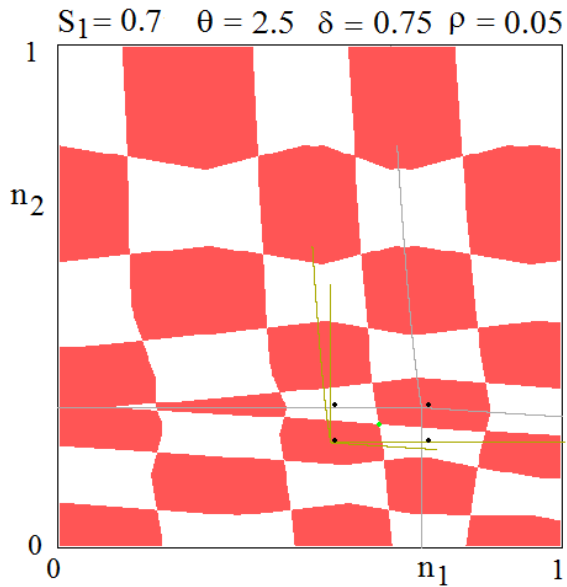


Red (Sync. 2-cycle) becomes dominant.

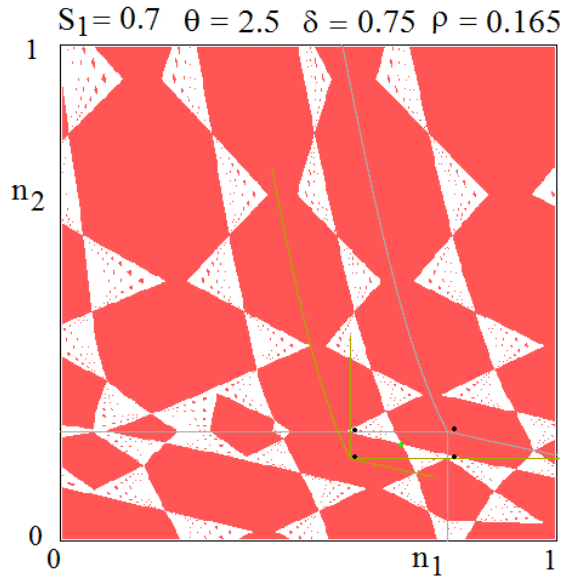
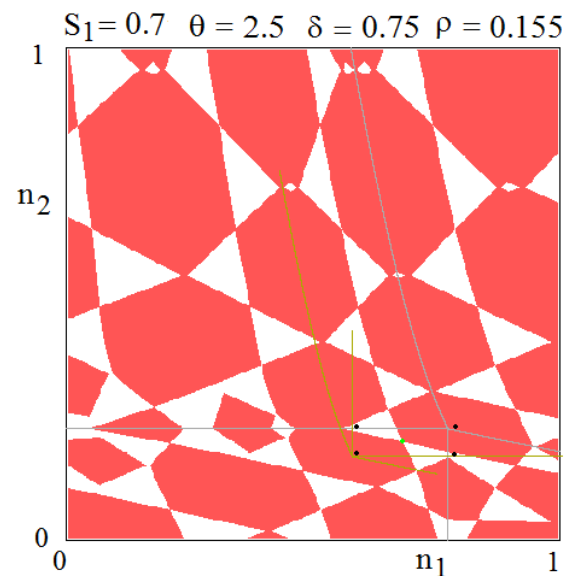


Sym. Async. 2-cycle becomes a node at  $\rho = .817867$ , a saddle at  $\rho = .833323$ .

**Asymmetric Synchronized & Asynchronized 2-Cycles**  $s_1 = 0.7, \theta = 2.5; \delta = 0.75$



By  $\rho = .165$ , infinitely many Red islands appear inside White.

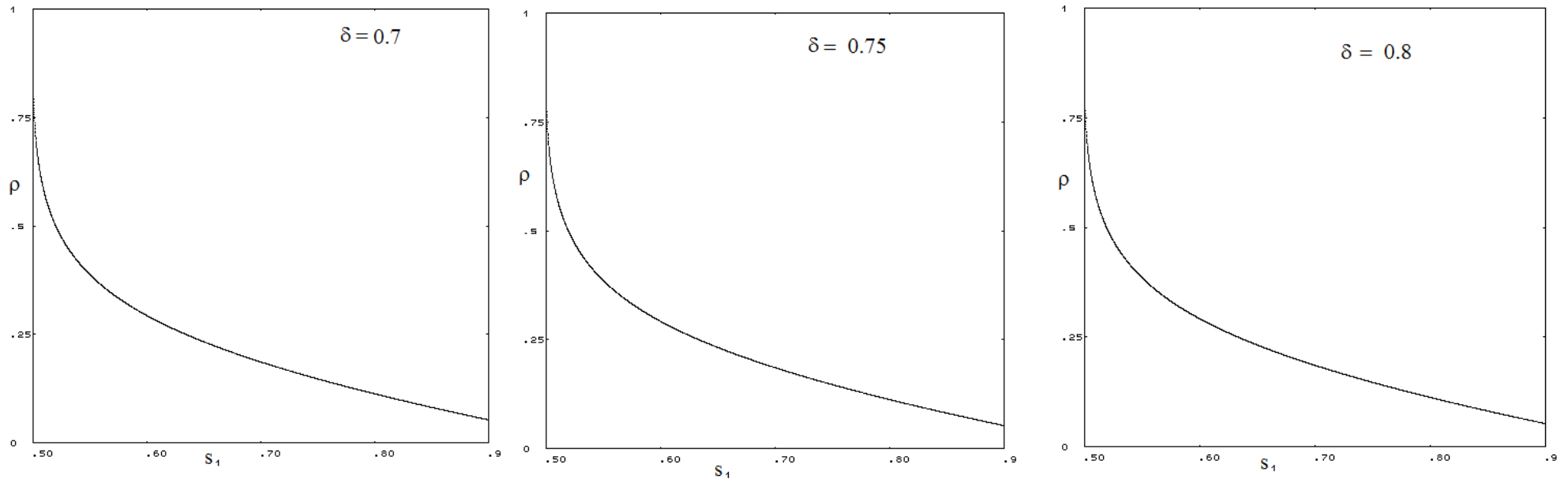


By  $\rho = .19$ , the stable asynchronized 2-cycle collides with its basin boundary and disappears, leaving the **Synchronized 2-cycle as the unique attractor.**

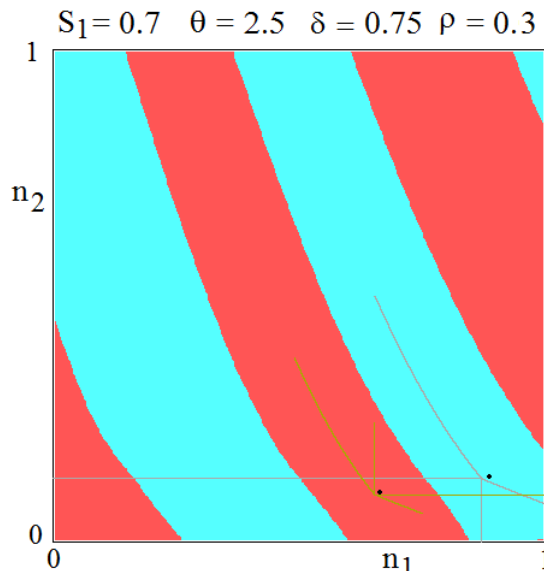
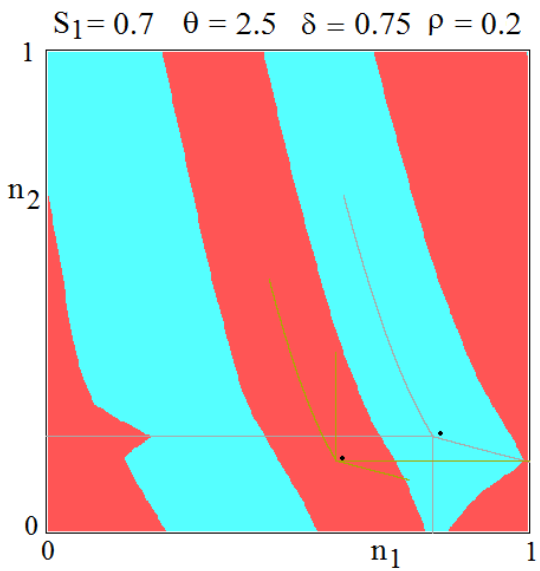
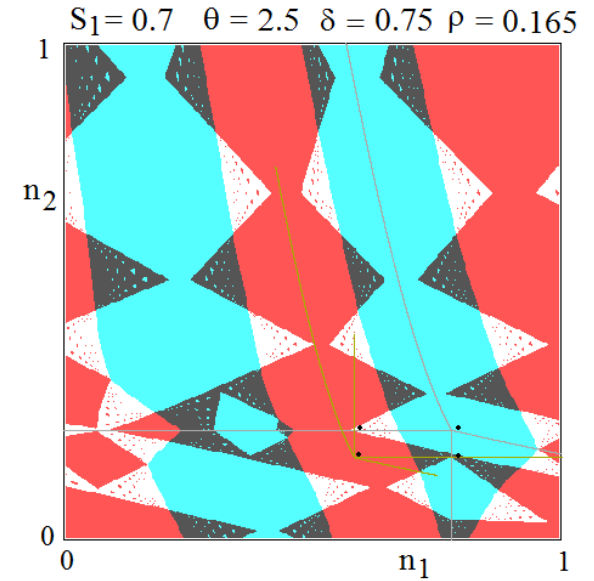
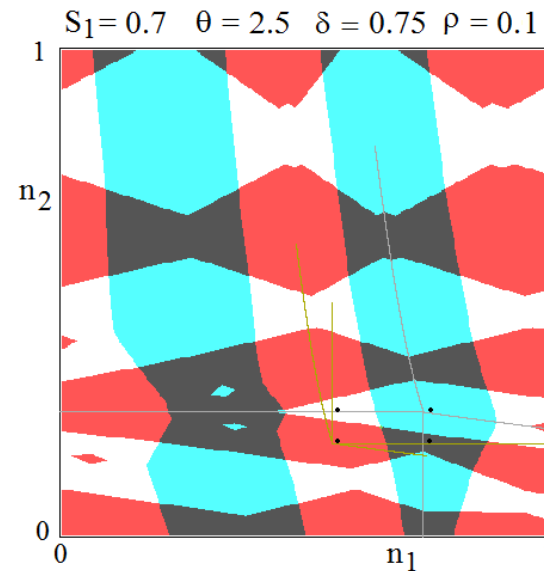
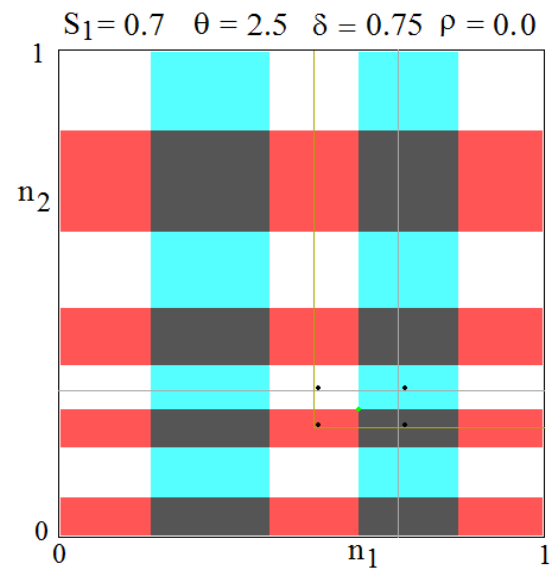
## A Smaller Reduction in Trade Costs Synchronizes Innovation Cycles with Country Size Asymmetries

**Critical Value of  $\rho_c$  at which the Stable Asynchronized 2-cycle disappears (as a function of  $s_1$ )**

- It declines very rapidly as  $s_1$  increases from 0.5.
- It hardly changes with  $\delta$ .



### Four Basins of Attraction ( $s_1 = 0.7, \theta = 2.5, \delta = 0.75$ )

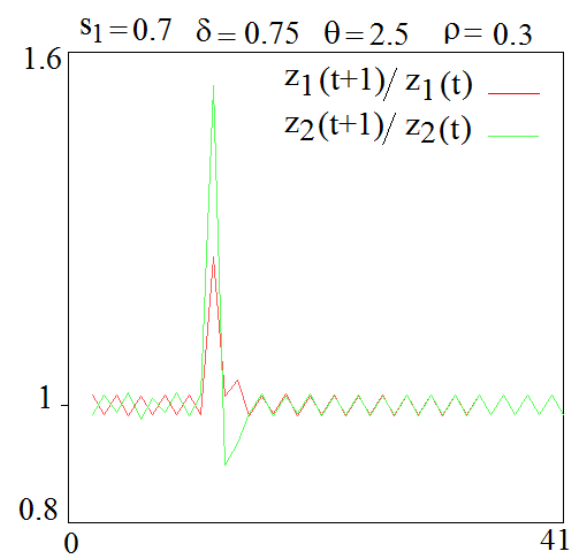
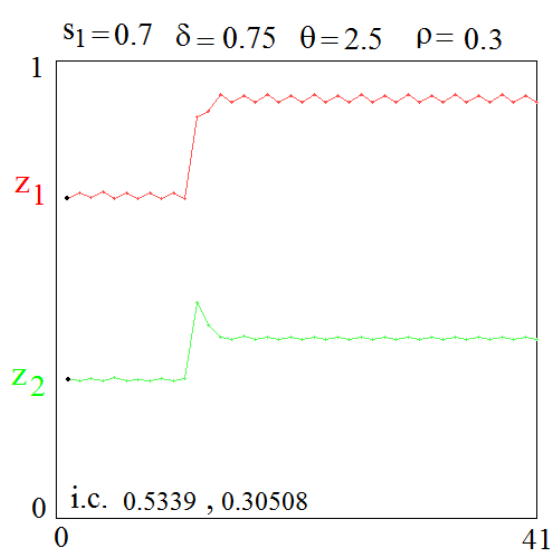
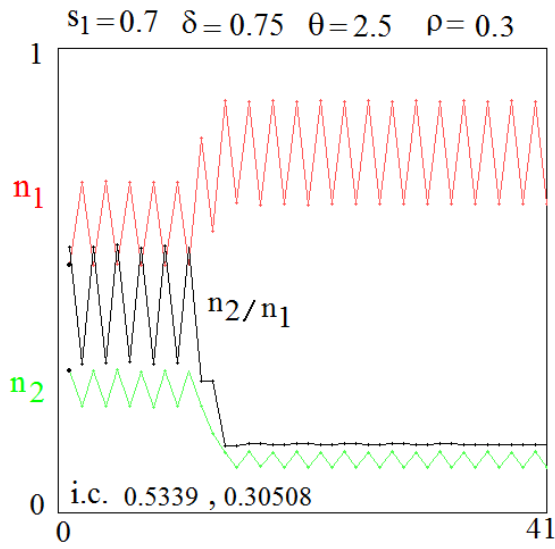
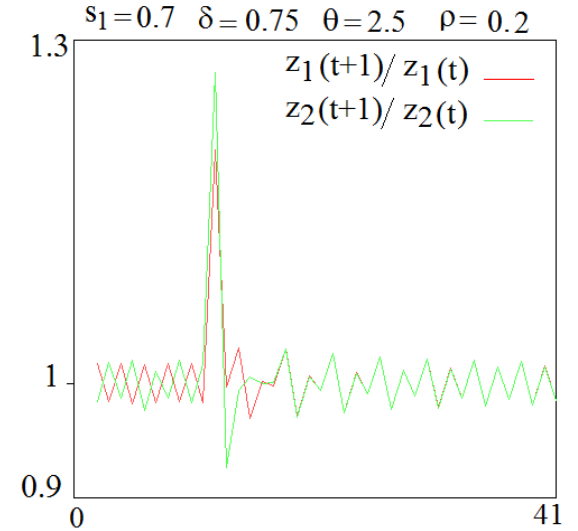
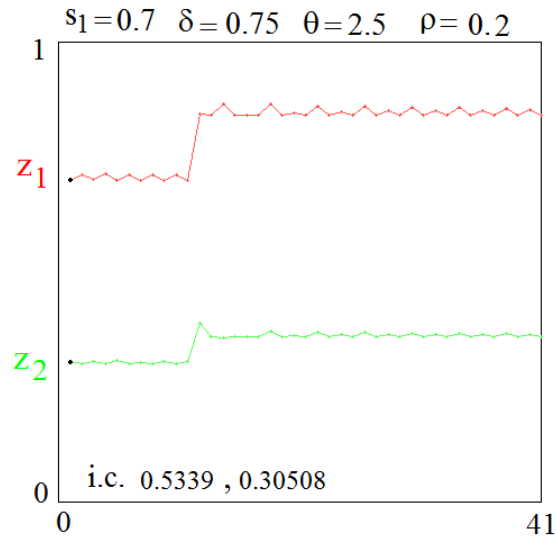
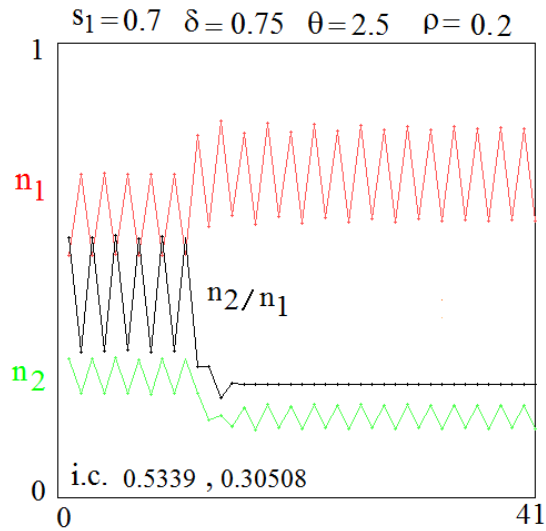


As  $\rho$  rises,  
 Red invades White, and Azure  
 invades Gray, and vertical  
 slips of Red and Azure  
 emerge.

### Three Effects of Globalization: Home Market Effect

### Productivity Gains

### Synchronization



## **Concluding Remarks**

## Summary:

- 1st attempt to explain why globalization might synchronize endogenous productivity fluctuations
- *Key Mechanism*: Globalization → Innovators from everywhere competing against each other in more integrated (hence common) market → Alignment of Incentives to Innovate → Synchronization
- Captured in a 2-country model of endogenous innovation cycles, built on DJ and HK
  - In autarky, innovation dynamics of the two countries are decoupled.
  - As trade cost falls and intra-industry trade rise, they become more synchronized.
  - Full synchronization at a larger trade cost with more unequal country sizes.
  - The smaller country adjusts its rhythm to the rhythm of the bigger country.
- Adding endogenous sources of fluctuations helps to understand “the trade-comovement puzzle.”
- Technical Contributions
  - 1<sup>st</sup> two-country model of endogenous fluctuations
  - A New Model of Coupled Oscillators
  - Application of 2D noninvertible, PWS, discrete time dynamic system



## Next Steps:

- **Synchronization of Chaotic Fluctuations**
- **Many Countries**
- **Different Models of Endogenous Innovation Cycles:**
  - *My conjecture:* Globalization should cause synchronization as long as it causes innovators based in different countries to operate in a common market environment.
  - The assumption of structural similarity seems crucial.

What if two countries are structurally dissimilar?

- **Different Models of Trade:** For example,
  - What if the two countries become vertically specialized?; e.g., global supply chains
  - Two Industries: **Upstream & Downstream**, each produces DS composite as in DJ.
  - One country has comparative advantage in **U**; the other in **D**
  - *My conjecture:* Globalization leads to an asynchronization

Empirically consistent, as the evidence for the synchronizing effect of trade is strong among developed countries, but *not so* btw developed and developing countries